DYNAMIC ANALYSIS OF NONSTEADY HEAT TRANSFER IN A FIXED BED OF SPHERES AT A VARIABLE GAS-FLOW TEMPERATURE

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The analysis of heat transfer is reduced to the solution of an ordinary first-order differential equation by simplifying the dynamic characteristics of the system.

A gas stream with a variable initial temperature is passed through a fixed bed of spheres in the direction h. The heat-transfer process is described by a system of differential equations [1, 2]

$$\frac{\partial t(r,h,\tau)}{\partial \tau} = a\left[\frac{\partial^2 t(r,h,\tau)}{\partial r^2} + \frac{2}{r}\frac{\partial t(r,h,\tau)}{\partial r}\right], \quad (1)$$

$$\frac{\partial t(h,\tau)}{\partial h} + \frac{1}{v}\frac{\partial t(h,\tau)}{\partial \tau} + \frac{\alpha FS}{vfc}[t(h,\tau) - t(R,h,\tau)] = 0 \quad (2)$$

for the boundary conditions

$$\lambda \left.\frac{\partial t(r,h,\tau)}{\partial r}\right|_{r=R} = \alpha[t(h,\tau) - t(R,h,\tau)], \quad (3)$$

$$\left.\frac{\partial t(r,h,\tau)}{\partial r}\right|_{r=0} = 0, \quad (4)$$

$$t(h,0) = t(r,h,0) = 0 \quad (0 \leqslant r \leqslant R;\ h \geqslant 0). \quad (5)$$

The gas temperature at the inlet section of the bed is $t(0, \tau)$, which is a specified function of time.

The solution of this system of equations, transformed after Laplace, has the form

$$t(h,p) = \exp\left\{k[W(R,p) - 1] - \frac{p}{v}\right\} t(0,p), \quad (6)$$

$$t(r,h,p) = W(r,p)\exp\left\{k[W(R,p) - 1] - \frac{p}{v}\right\} t(0,p), \quad (7)$$

where

$$k = \frac{\alpha FSh}{vfc};$$

$$W(r,p) = \frac{t(r,h,p)}{t(h,p)} = \frac{R}{r} \times \frac{\mathrm{Bi}\,\mathrm{sh}\sqrt{\frac{p}{a}}\,r}{(\mathrm{Bi}-1)\,\mathrm{sh}\sqrt{\frac{p}{a}}\,R + \sqrt{\frac{p}{a}}\,R\,\mathrm{ch}\sqrt{\frac{p}{a}}\,R};$$

$$W(R,p) = \frac{t(R,h,p)}{t(h,p)} = W(r,p)|_{r=R}.$$

For the average temperature of the sphere we have

$$t_{av}(p) = W_{av}(p)\exp\left\{k[W(R,p) - 1] - \frac{p}{v}\right\} t(0,p), \quad (8)$$

where

$$W_{av}(p) = \frac{t_{av}(p)}{t(h,p)} = \frac{3\,\mathrm{Bi}\left(\sqrt{\frac{p}{a}}\,R\,\mathrm{ch}\sqrt{\frac{p}{a}}\,R - \mathrm{sh}\sqrt{\frac{p}{a}}\,R\right)}{p\frac{R^2}{a}\left[(\mathrm{Bi}-1)\,\mathrm{sh}\sqrt{\frac{p}{a}}\,R + \sqrt{\frac{p}{a}}\,R\,\mathrm{ch}\sqrt{\frac{p}{a}}\,R\right]}$$

The quantity p/v in expressions (6)–(8) can be neglected, since it accounts for the time in which the gas is passed through the bed. This time is generally negligibly small in comparison with the other time constants of the system. Thus

$$t(h,p) = W_1(p)t(0,p), \quad (9)$$

$$t(r,h,p) = W_2(p)t(0,p), \quad (10)$$

$$t_{av}(p) = W_3(p)t(0,p), \quad (11)$$

where

$$W_1(p) = \exp\{k[W(R,p) - 1]\};$$

$$W_2(p) = W_1(p)\,W(r,p); \quad W_3(p) = W_1(p)\,W_{av}(p).$$

Substituting $p = j\omega$ into the transfer functions $W_1(p)$, $W_2(p)$, or $W_3(p)$, we obtain the amplitude-phase frequency characteristics (AFC) of the system:

$$W(j\omega) = A(\omega)\exp[j\varphi(\omega)] = \mathrm{Re}(\omega) + j\,\mathrm{Im}(\omega). \quad (12)$$

If the input quantity $t(0, \tau)$ oscillates harmonically with frequency ω, expression (12) immediately yields the amplitude $A(\omega)$ and the phase shift $\varphi(\omega)$ of the output quantities $t(h, \tau)$, $t(r, h, \tau)$, or $t_{av}(\tau)$ in the quasi-steady regime. In this case there is no need to return to the preimages in (9)–(11).

With a nonharmonic variation in $t(0, \tau)$, the transition to the preimages in (9)–(11) is an exceedingly difficult operation; we will therefore simplify the transfer functions $W_1(p)$, $W(r, p)$, and $W_{av}(p)$.

Figure 1 shows a family of $W_1(j\omega)$ hodographs for various values of Bi and k. For $k < 2\mathrm{Bi}/5 + 2$ a portion of the hodographs can be approximated by the expression

$$W(j\omega) = \frac{jT_1\omega + 1}{jT_2\omega + 1},\ T_1 > 0 \quad (13)$$

(curves 1–4), and a portion can be approximated for $k < 2\mathrm{Bi}/5 + 2$ by the expression

$$W(j\omega) = \frac{\exp(-j\tau_l\omega)}{jT\omega + 1}, \quad \tau_l > 0 \tag{14}$$

(curves 5–7).

Requiring coincidence of the hodographs $W(j\omega)$ and $W_1(j\omega)$ as $\omega \to 0$ we derive a system of equations for the determination of the time constants T_1, T_2, T, and τ_3 of the approximating expressions (13) and (14):

$$\left.\begin{aligned} T_2 - T_1 &= T + \tau_l = \lim_{p\to 0}\frac{1 - W_1(p)}{p}, \\ T_2^2 - T_1^2 &= T^2 = \lim_{\omega\to 0}\frac{1 - |W_1(j\omega)|^2}{\omega^2}. \end{aligned}\right\} \tag{15}$$

An ordinary differential first-order equation with a lag argument

$$T\,\frac{dt_{\mathrm{out}}(\tau)}{d\tau} + t_{\mathrm{out}}(\tau) = t_{\mathrm{in}}(\tau - \tau_3) \tag{16}$$

corresponds to the frequency characteristic (14) and is easily solved analytically or graphically for any

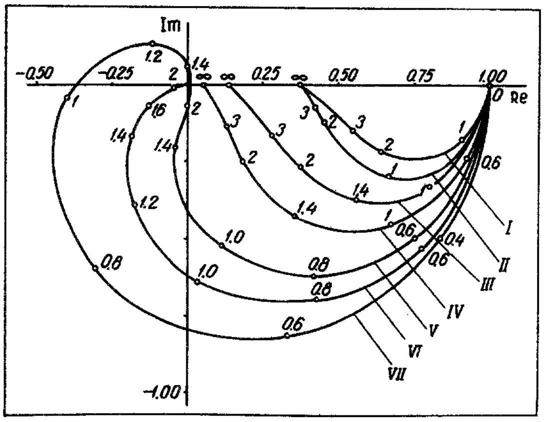

Fig. 1. Amplitude-phase frequency characteristics of the system within the channel for transmission of the effect of the gas temperature $t(0, \tau)$ I) at Bi = 3; k = 1; II) 1 and 1, respectively; III) 3 and 2; IV) 2 and 5; VI) 4 and 10; VII) 4 and 20; figures at points, values of parameter $x = \sqrt{(\omega/2a)}R$.

function $t_{\mathrm{in}}(\tau) = t(0, \tau)$ [3]. We solve the differential equation

$$T_2\frac{dt_{\mathrm{out}}(\tau)}{d\tau} + t_{\mathrm{out}}(\tau) = T_1\frac{dt_{\mathrm{in}}(\tau)}{d\tau} + t_{\mathrm{in}}(\tau) \tag{17}$$

as easily, this equation corresponding to the approximating AFC (13).

Figure 2 shows the AFC hodographs for the sphere-temperature transfer functions $W(R, p)$, $W_{\mathrm{av}}(p)$, and

$$W(0, p) = \frac{t(0, h, p)}{t(h, p)} = W(r, p)|_{r=0}.$$

The $W(R, j\omega)$ and $W_{\mathrm{av}}(j\omega)$ hodographs coincide satisfactorily in form with the approximating AFC (13), while the $W(0, j\omega)$ hodograph coincides satisfactorily with AFC (14).

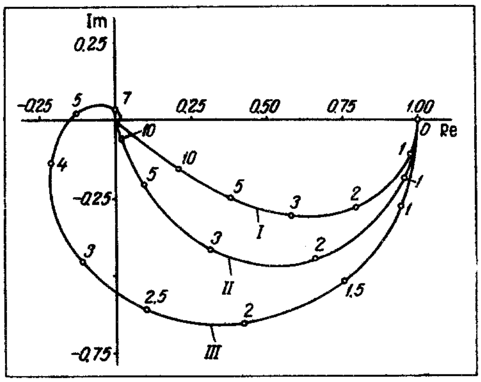

Fig. 2. Amplitude-phase frequency characteristics of system within the channel for transmission of the effect of the gas temperature $t(h, \tau)$ and the sphere temperature at Bi = 3; I) surface temperature $t(R, h, \tau)$; II) mean temperature $t_{\mathrm{av}}(\tau)$; III) center temperature $t(0, h, \tau)$; figures at points, values of parameter $x_1 = \sqrt{(\omega/a)}R$.

The time constants T_2, T_1, T, and in this case are also determined from system (15) in which insteady of $W_1(p)$ and $W_1(j\omega)$ we have the transfer functions and the AFC of the sphere temperature: $W(R, p)$ and $W(R, j, \omega)$, $W_{\mathrm{av}}(p)$ and $W_{\mathrm{av}}(j\omega)$, and $W(0, p)$ and $W(0, j\omega)$.

We will illustrate the application of the dynamic-analysis method with the following example:

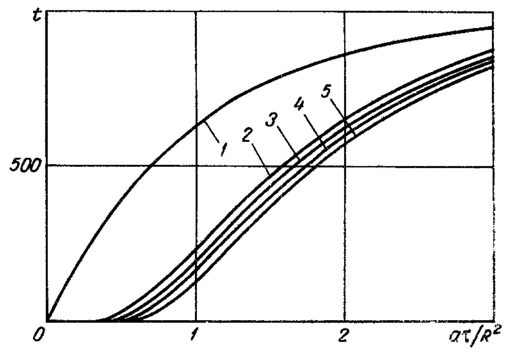

Fig. 3. Temperature curves (example): 1) gas temperature $t(0, \tau)$, °C; in input section of bed 2) $t(h, \tau)$; 3) $t(R, h, \tau)$; 4) $t_{\mathrm{av}}(\tau)$; 5) $t(0, h, \tau)$.

Example. Construct the curves $t(h, \tau)$, $t(R, h, \tau)$, $t_{\mathrm{av}}(\tau)$, and $t(0, h, \tau)$ when k = 10 and Bi = 4 and for the exponential function $t(0, \tau)$

$$t(0, \tau) = 1000\left[1 - \exp\left(-\frac{a\tau}{R^2}\right)\right], \ ^\circ\mathrm{C}. \tag{a}$$

Solution. Let us subject the input quantity (a) to Laplace transformation:

$$t(0, p) = \frac{1000}{p\left(\frac{R^2}{a}p + 1\right)}, \quad \text{(b)}$$

so that the images of the output quantities are written as follows:

the gas temperature

$$t(h, p) = W(p)\, t(0, p), \quad \text{(c)}$$

where

$$W(p) = \frac{\exp(-\tau_l p)}{Tp + 1} \quad [\sec(14)];$$

$$\left.\begin{aligned} T + \tau_l &= \frac{k}{3\,\mathrm{Bi}} \frac{R^2}{a} \\ T^2 &= 2k\left(\frac{1}{45\,\mathrm{Bi}} + \frac{1}{9\,\mathrm{Bi}^2}\right)\frac{R^4}{a^2} \end{aligned}\right\} [\sec(15)];$$

$$T = \frac{R^2}{a}\sqrt{2k\left(\frac{1}{45\,\mathrm{Bi}} + \frac{1}{9\,\mathrm{Bi}^2}\right)} = 0.500\,\frac{R^2}{a};$$

$$\tau_l = \left[\frac{k}{3\,\mathrm{Bi}} - \sqrt{2k\left(\frac{1}{45\,\mathrm{Bi}} + \frac{1}{9\,\mathrm{Bi}^2}\right)}\right]\frac{R^2}{a} = 0.333\,\frac{R^2}{a};$$

the temperature of the sphere surface

$$t(R, h, p) = W(p)\, t(h, p), \quad \text{(d)}$$

where

$$W(p) = \frac{T_1 p + 1}{T_2 p + 1} \quad [\sec(13)];$$

$$T_2 - T_1 = \frac{1}{3\,\mathrm{Bi}}\frac{R^2}{a};$$

$$T_2 = \left(\frac{1}{15} + \frac{1}{3\,\mathrm{Bi}}\right)\frac{R^2}{a} = 0.150\,\frac{R^2}{a};$$

$$T_1 = \frac{1}{15}\frac{R^2}{a} = 0.067\,\frac{R^2}{a};$$

the average sphere temperature

$$t_{av}(p) = W(p)\, t(h, p), \quad \text{(e)}$$

where

$$W(p) = \frac{T_1 p + 1}{T_2 p + 1};$$

$$T_2 - T_1 = \left(\frac{1}{15} + \frac{1}{3\,\mathrm{Bi}}\right)\frac{R^2}{a};$$

$$T_2 = \frac{1}{\mathrm{Bi} + 5}\left(\frac{2\mathrm{Bi}}{21} + \frac{2}{3} + \frac{5}{3\,\mathrm{Bi}}\right)\frac{R^2}{a} = 0.163\,\frac{R^2}{a};$$

$$T_1 = \frac{1}{\mathrm{Bi} + 5}\frac{\mathrm{Bi}}{35}\frac{R^2}{a} = 0.013\,\frac{R^2}{a};$$

the temperature of the sphere center

$$t(0, h, p) = W(p)\, t(h, p), \quad \text{(f)}$$

where

$$W(p) = \frac{\exp(-\tau_l p)}{Tp + 1};$$

$$T + \tau_l = \left(\frac{1}{6} + \frac{1}{3\,\mathrm{Bi}}\right)\frac{R^2}{a};$$

$$T = \frac{R^2}{a}\sqrt{\frac{1}{90} + \frac{2}{45\,\mathrm{Bi}} + \frac{1}{9\,\mathrm{Bi}^2}} = 0.171\,\frac{R^2}{a};$$

$$\tau_l = \left(\frac{1}{6} + \frac{1}{3\,\mathrm{Bi}} - \sqrt{\frac{1}{90} + \frac{2}{45\,\mathrm{Bi}} + \frac{1}{9\,\mathrm{Bi}^2}}\right)\frac{R^2}{a} = 0.079\,\frac{R^2}{a}.$$

Turning to the preimages in (c)–(f), we find the unknown output quantities. These are shown in Fig. 3.

NOTATION

τ is the time; h is the bed coordinate in the direction of gas flow; r is the point coordinate inside the sphere; R is the sphere radius; $t(r, h, \tau)$ is the temperature of the sphere point; $t_{av}(\tau)$ is the mean integral temperature of the sphere; $t(h, \tau)$ is the gas temperature; for the sphere material, a is the thermal diffusivity; λ is the thermal conductivity; for the gas flow c is the volumetric heat capacity; f is the area of effective cross section; v is the velocity; α is the coefficient of heat transfer from gas to sphere surfaces; F is the sphere surface per unit volume of bed; S is the area of the bed section; k is a parameter; Bi is the Biot number; p is the parameter of the Laplace transform; t(p) is the Laplace transformation of $t(\tau)$; W(p) is the transfer function; $W(j\omega)$ is the amplitude-phase frequency characteristic; $j = \sqrt{-1}$; ω is the angular frequency; τ_l is the time lag; T and T_2 are time constants of the aperiodic link; T_1 is the time constant of the forcing link; t_{in} is the input value; t_{out} is the output value; $\mathrm{Im}(\omega)$ is the imaginary frequency characteristic; $\mathrm{Re}(\omega)$ is the material frequency characteristic.

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